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Analytical Formulation of M-N Domain for Rectangular R.C. Cross Sections with Asymmetrical Reinforcement

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INTRODUCTION

The ultimate limit state check of reinforced concrete cross sections under axial force and bending moment is often performed using M_{Rd}-N_{Rd} interaction curves. The exact evaluation of such curves according to the code provisions [1, 3, 4] is cumbersome and requires proper computer programs. Many authors have proposed different methods and approaches in order to evaluate approximate interaction domains for reinforced concrete sections, as in detail described in [2]. All these methods refer to symmetrically reinforced cross sections, because this is the most common situation in the design (e.g., oscillation during a seismic event causes reverted bending moments, thus making necessary equal reinforcement at opposite sides of the cross section). Nevertheless there are particular cases in which the design bending moment has a specific sign, and in these cases the possibility of an asymmetrical disposition of reinforcement has to be considered. This paper, starting from previous studies and formulations given by the authors [5, 6], points out specific aspects of M-N interaction curves for rectangular cross sections with asymmetrical reinforcement and proposes an analytical formulation, based on few parameters (maximum axial and bending resistance, separately evaluated for concrete and steel; axial and bending resistance variation due to asymmetry in reinforcement), which accurately describes the exact interaction curves for rectangular cross sections both with symmetrical and asymmetrical reinforcement. An attempt to revert check approach in order to provide design formulations is finally performed.

Keywords: concrete, M-N interaction curves, design

EVALUATION OF M-N INTERACTION CURVE

The interaction curve is the set of M-N couples corresponding to strain diagrams which reach (and not pass) the ultimate strain value. Assuming as positive elongation for strain and tension for stress, but regarding as sign-less the design strength f_{yd} and f_{cd}, according to the last version of Eurocode 2 provisions [4] the stress-strain relationship of steel (Fig. 1A) is given by

$$\begin{aligned} &\sigma_s = -f_{yd} && \text{for} \quad \varepsilon \le -\varepsilon_{yd} \\ &\sigma_s = E_s\,\varepsilon && \text{for} \quad -\varepsilon_{yd} \le \varepsilon \le \varepsilon_{yd} \\ &\sigma_s = f_{yd} && \text{for} \quad \varepsilon \ge \varepsilon_{yd} \end{aligned} \tag{1}$$

without any strain limit. The stress-strain relationship of concrete (Fig. 1B) is given by

$$\begin{aligned} &\sigma_c = -f_{cd}\left[1-\left(1-\frac{\varepsilon_c}{\varepsilon_{c2}}\right)^n\right] && \text{for} \quad \varepsilon_{c2} \le \varepsilon \le 0 \\ &\sigma_c = -f_{cd} && \text{for} \quad \varepsilon_{cu2} \le \varepsilon \le \varepsilon_{c2} \end{aligned} \tag{2}$$

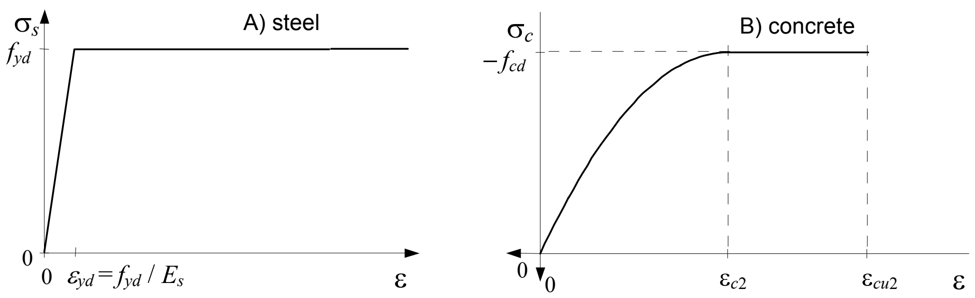

Fig. 1. Stress-strain relationship for steel and concrete.

The values of ε_{c2}, ε_{cu} and n depend on the strength class of concrete (if $f_{ck} \leq 50$ MPa, it is $\varepsilon_{c2} = -2.0 \times 10^{-3}$, $\varepsilon_{cu2} = -3.5 \times 10^{-3}$, $n = 2.0$).
If the cross section is partially compressed, the compressive strain in the concrete is limited to ε_{cu2}. In cross sections subjected to concentric loading the limiting compressive strain is assumed to be ε_{c2}. In intermediate situations of fully compressed cross sections, a limiting value ε_{c2} is assumed at a distance $(1 - \varepsilon_{c2} / \varepsilon_{cu2})\, h$ from the more compressed edge.
The possible range of strain distributions is shown in Fig. 2. The figure points out some peculiar strain diagrams, corresponding to the yielding of upper reinforcement in tension (C'$_{s+}$) and in compression (C'$_{s-}$), the yielding of lower reinforcement in tension (C'), the transition from partially compressed to fully compressed cross section (D), the yielding of lower reinforcement in compression (D') and the concentric loading in compression of the cross section (E). The diagrams C'$_{s+}$ C'$_{s-}$ and C' correspond to a neutral axis respectively located at a distance from the compressed edge of the cross section

$$\xi_1 = \frac{\varepsilon_{cu2}}{\varepsilon_{cu2} - \varepsilon_{yd}}\gamma \qquad \xi_2 = \frac{\varepsilon_{cu2}}{\varepsilon_{cu2} + \varepsilon_{yd}}\gamma \qquad \xi_3 = \frac{\varepsilon_{cu2}}{\varepsilon_{cu2} - \varepsilon_{yd}}(1-\gamma) \tag{3}$$

The diagram D' corresponds to a strain $\varepsilon_{c,\min}$ at the lower edge of the fully compressed cross section

$$\varepsilon_{c,\min} = \varepsilon_{c2}\,\frac{\varepsilon_{cu2}\;\gamma + \varepsilon_{yd}}{\varepsilon_{cu2}\;\gamma - \varepsilon_{c2}} = \varepsilon_{c2}\;\eta_{\min(D')} \tag{4}$$

For any given strain diagram which reaches the ultimate strain value it is possible to evaluate the corresponding stress diagrams both for concrete and steel reinforcements and the corresponding resultant forces (Fig. 3). It is thus possible to evaluate the normalised axial force ν and bending moment μ acting in the cross section

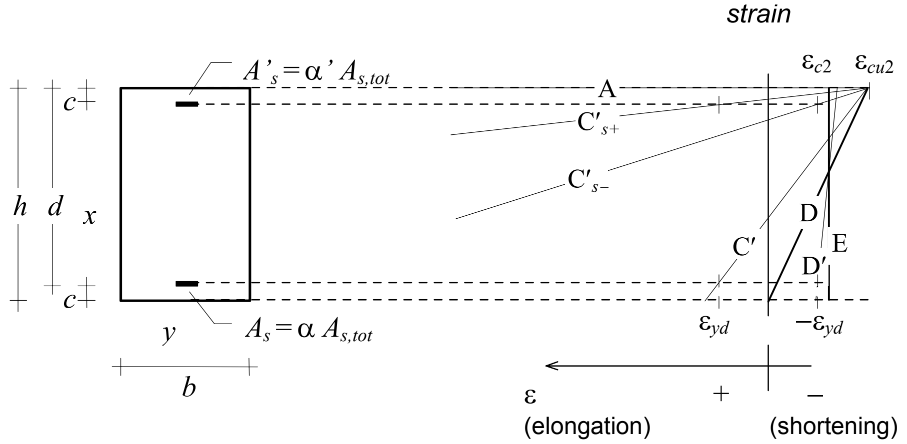

Fig. 2. Possible strain distributions.

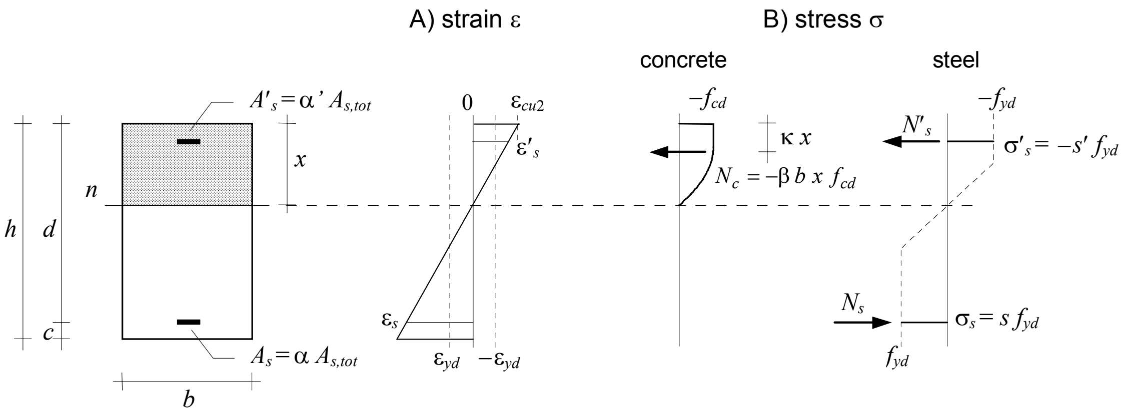

Fig. 3. Strain and stress diagrams; resultant of stresses in concrete and steel.

$$\begin{aligned} \nu &= \beta\,\xi + \omega_{tot}\,(s'\,\alpha' - s\,\alpha) \\ \mu &= \beta\,\xi\,(0.5-\kappa\,\xi) + \omega_{tot}\,(s'\,\alpha' + s\,\alpha)\,(0.5-\gamma) \end{aligned} \tag{5}$$

The coefficients β, κ, s, s' depend on the strain diagram, as shown in Tab. 1. By varying ξ from 0 to 1 and η_{min} from 0 to 1, equations (5) provide the portion of interaction curve corresponding to the reaching of maximum compressive strain at the top of the cross section. The remaining part, corresponding to maximum compressive strain at the bottom of the cross section, is obtained by inverting α and α' and changing sign to μ.

Tab. 1. Coefficients β, κ, s, s' (Note: values and expressions of β and κ are referred to $f_{ck} \le 50$ MPa).

ξ	η_{min}	β	κ	s	s'
$0 \le \xi \le \xi_1$	-	0.810	0.416	1	-1
$\xi_1 \le \xi \le \xi_2$	-	0.810	0.416	1	$-\dfrac{\varepsilon_{cu2}}{\varepsilon_{yd}}\dfrac{\xi-\gamma}{\xi}$
$\xi_2 \le \xi \le \xi_3$	-	0.810	0.416	1	1
$\xi_3 \le \xi \le 1$	-	0.810	0.416	$-\dfrac{\varepsilon_{cu2}}{\varepsilon_{yd}}\dfrac{1-\gamma-\xi}{\xi}$	1
1	$0 \le \eta_{min} \le \eta_{min\,(D')}$	$1-\dfrac{4}{21}(1-\eta_{min})^2$	$\dfrac{1}{2}\dfrac{1-\frac{16}{49}(1-\eta_{min})^2}{1-\frac{4}{21}(1-\eta_{min})^2}$	$\dfrac{\varepsilon_{c2}}{\varepsilon_{yd}}\eta_{min} + \dfrac{\varepsilon_{cu2}}{\varepsilon_{yd}}(1-\eta_{min})\gamma$	1
1	$\eta_{min\,(D')} \le \eta_{min} \le 1$	$1-\dfrac{4}{21}(1-\eta_{min})^2$	$\dfrac{1}{2}\dfrac{1-\frac{16}{49}(1-\eta_{min})^2}{1-\frac{4}{21}(1-\eta_{min})^2}$	-1	1

CONSIDERATIONS ON M-N INTERACTION CURVE

Non-reinforced cross sections

If no reinforcement is present in the cross section (i.e., ω_{tot} = 0) equations (5) may be unified in

$$\mu = \nu\left(0.5 + \frac{\kappa}{\beta}\nu\right) \tag{6}$$

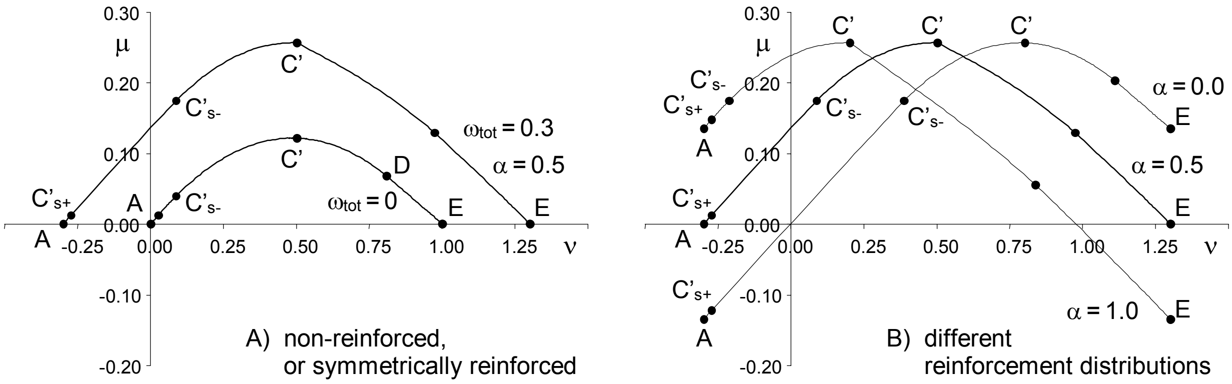

Fig. 4. Interaction curves for a non-reinforced (ω_{tot} = 0) and reinforced (ω_{tot} = 0.3) cross section, with f_{yk} = 430 MPa, γ = 0.05.

Until the cross section is partially compressed (i.e., from A to D) β and κ are constant and equation (6) represents a parabola (Fig. 4A). Its maximum is achieved when ξ = 1 / 4 κ ≅ 0.6 (i.e., in most cases close to the point C') and corresponds to the couple

$$\nu = \nu_M = \frac{\beta}{4\,\kappa}$$
$$\mu = \mu_{c,\max} = \frac{\beta}{16\,\kappa} \tag{7}$$

When the cross section is fully compressed, using the expressions of β and κ referred in Tab. 1 we obtain

$$\mu = \frac{6}{7}\nu^3 - \frac{5}{14}\nu^2 + \frac{1}{2}\nu \tag{8}$$

The final branch of the interaction curve (which ends at ν = 1) is thus a third-degree curve, but it is not too much different from the previous parabola.

Symmetrically reinforced cross sections

When a symmetrical reinforcement is present (α = α' = 0.5), within the strain distributions ranging from A to C'$_{s+}$ (for which it is s = 1, s' = –1) equations (5) become

$$\nu = \beta\,\xi - \omega_{tot}$$
$$\mu = \beta\,\xi\,(0.5 - \kappa\,\xi) + \omega_{tot}\,(\alpha - \alpha')\,(0.5 - \gamma) \tag{9}$$

thus showing a quadratic relation between μ and ν. Anyway, the portion A – C'$_{s+}$ of the curve is usually very short and it may be relevant only when the normalised cover γ is particularly high.
More significantly, manipulating equations (5) so as to eliminate s' we obtain

$$\mu = (\nu + 2\,\omega_{tot}\,s\,\alpha)\,(0.5 - \gamma) + \beta\,\xi\,(\gamma - \kappa\,\xi) \tag{10}$$

In this expression, the last term is almost negligible within the strain distributions ranging from A to C'$_{s-}$, because both factors ξ and (γ – κ ξ) are small. The first portion of the interaction curve is thus approximately linear, as it is confirmed by Fig. 4A. Its starting point corresponds to the tensile strength of reinforcement

$$\nu = -\omega_{tot} \tag{11}$$

On the other side, within the strain distributions ranging from C'$_{s-}$ to C' (for which it is s = s' = 1) equations (5) become

$$\begin{aligned}\nu &= \beta\,\xi \\ \mu &= \beta\,\xi\,(0.5-\kappa\,\xi)+\omega_{tot}\;(0.5-\gamma)\end{aligned} \tag{12}$$

thus giving

$$\mu = \nu\left(0.5+\frac{\kappa}{\beta}\,\nu\right)+\omega_{tot}\;(0.5-\gamma) \tag{13}$$

This means that the branch C'_{s-} – C' of the interaction curve (Fig. 4A) is vertically translated, with respect to that of the non-reinforced cross section, of a quantity corresponding to the maximum bending contribution of reinforcements

$$\mu_{s,\max} = \omega_{tot}\;(0.5-\gamma) \tag{14}$$

Finally, it has to be noted that the last branch of the interaction curve, C' – E, which ends at $\nu = 1 + \omega_{tot}$, is more flat, with respect to a parabola, and this tendency grows up as the mechanical reinforcement ratio increases.

Asymmetrically reinforced cross sections

When the reinforcement distribution is not symmetrical ($\alpha \neq \alpha'$, but $\alpha + \alpha' = 1$), the M-N couple corresponding to pure tension (ξ=0, s =1, s' = –1) is

$$\begin{aligned}\nu &= -\omega_{tot} \\ \mu &= \Delta\mu_s = \omega_{tot}\;(\alpha'-\alpha)\,(0.5-\gamma)\end{aligned} \tag{15}$$

i.e., it corresponds to the same axial force as a symmetrically reinforced cross section, together with a non null bending moment (positive when $\alpha' > \alpha$). Equation (10) and the related considerations still apply, and the branch A – C'_{s-} of the interaction curve is once again approximately linear (although its ending point depends on α), as confirmed by Fig. 4B.
Within the strain distributions ranging from C'_{s-} to C' (for which it is $s = s' = 1$) equations (5) become

$$\begin{aligned}\nu &= \beta\,\xi+\omega_{tot}\;(\alpha'-\alpha) \\ \mu &= \beta\,\xi\,(0.5-\kappa\,\xi)+\omega_{tot}\;(0.5-\gamma)\end{aligned} \tag{16}$$

Comparing these with equations (12), it is apparent that the same values of μ are obtained, independently from α but corresponding to different values of ν, i.e., the branch C'_{s-} – C' of the interaction curve (Fig. 4B) is horizontally translated of a quantity depending on the difference of reinforcements

$$\Delta\nu_s = \omega_{tot}\;(\alpha'-\alpha) \tag{17}$$

Finally, the last branch of the interaction curve, C' – E, which ends once again at $\nu = 1 + \omega_{tot}$, is more or less flat, if compared to the symmetrically reinforced cross section, depending on the difference $\alpha' - \alpha$, i.e. on $\Delta\nu_s$.

PROPOSED FORMULATION OF M-N INTERACTION CURVE

Basing on the considerations carried on in the previous section, it is possible to consider the M-N interaction curve as constituted by three parts (linear, quadratic and curvilinear), described by the following equations

– for $-\omega_{tot} \leq \nu \leq \Delta\nu_s$

$$\mu = \mu_{s,\max}\left(1+\frac{\nu}{\omega_{tot}}\right)-\Delta\mu_s \tag{18A}$$

– for $\Delta\nu_s \le \nu \le \nu_M + \Delta\nu_s$

$$\mu = \mu_{c,\max} \left[1 - \left(\frac{\nu_M + \Delta\nu_s - \nu}{\nu_M} \right)^2 \right] + \mu_{s,\max} \tag{18B}$$

– for $\nu_M + \Delta\nu_s \le \nu \le 1 + \omega_{tot}$

$$\mu = (\mu_{c,\max} + \mu_{s,\max} - \Delta\mu_s) \left[1 - \left(\frac{\nu - \nu_M - \Delta\nu_s}{1 + \omega_{tot} - \nu_M - \Delta\nu_s} \right)^n \right] + \Delta\mu_s \tag{18C}$$

being $n = 1 + \left(\dfrac{\nu_M}{1 - \nu_M + \omega_{tot} - \Delta\nu_s} \right)^2$

The equations depend only on few parameters, already defined in the previous section and recalled in Tab. 2.

Tab. 2. Parameters which rule the proposed formulation.

Concrete		Steel	
ν_M	$= \dfrac{\beta}{4\,\kappa}$	$\Delta\nu_s$	$= \omega_{tot}\ (\alpha' - \alpha)$
$\mu_{c,\max}$	$= \dfrac{\beta}{16\,\kappa}$	$\mu_{s,\max}$	$= \omega_{tot}\ (0.5 - \gamma)$
		$\Delta\mu_s$	$= \omega_{tot}\ (\alpha' - \alpha)\,(0.5 - \gamma)$

Fig. 5 points out the three parts which constitute the curve. The portion of the M-N interaction curve corresponding to compression in the lower part of the cross section will be evaluated with the same equations, applied inverting α and α' and changing sign to μ, as shown in the same figure.
The results obtained with the proposed formulation are compared to the exact values in Fig. 6. When the normalised cover γ is small (e.g., γ=0.05), the differences are so negligible to make almost impossible to distinguish exact and proposed curves. Higher values of γ (e.g., γ=0.15), corresponding to sections having a very small depth or an unusually high concrete cover, reveal some differences, which are nevertheless very small in percentage and negligible in practical applications.

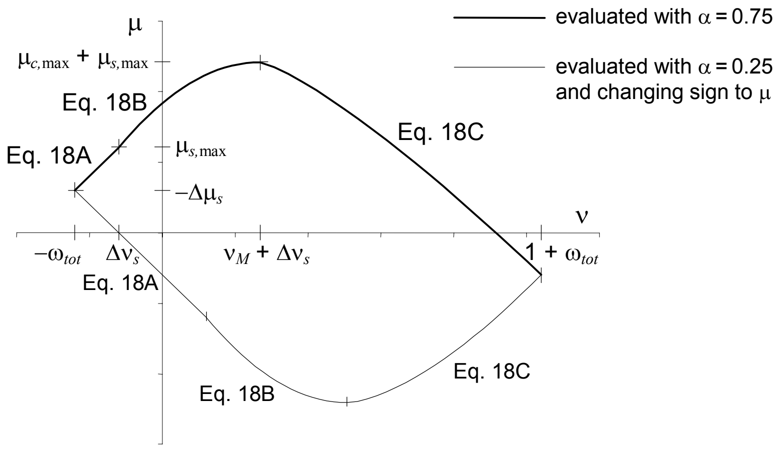

Fig. 5. Interaction curve for asymmetrically reinforced cross section according to the proposed formulation (f_{yk} = 430 MPa, γ = 0.1, ω_{tot} = 0.3, α=0.75).

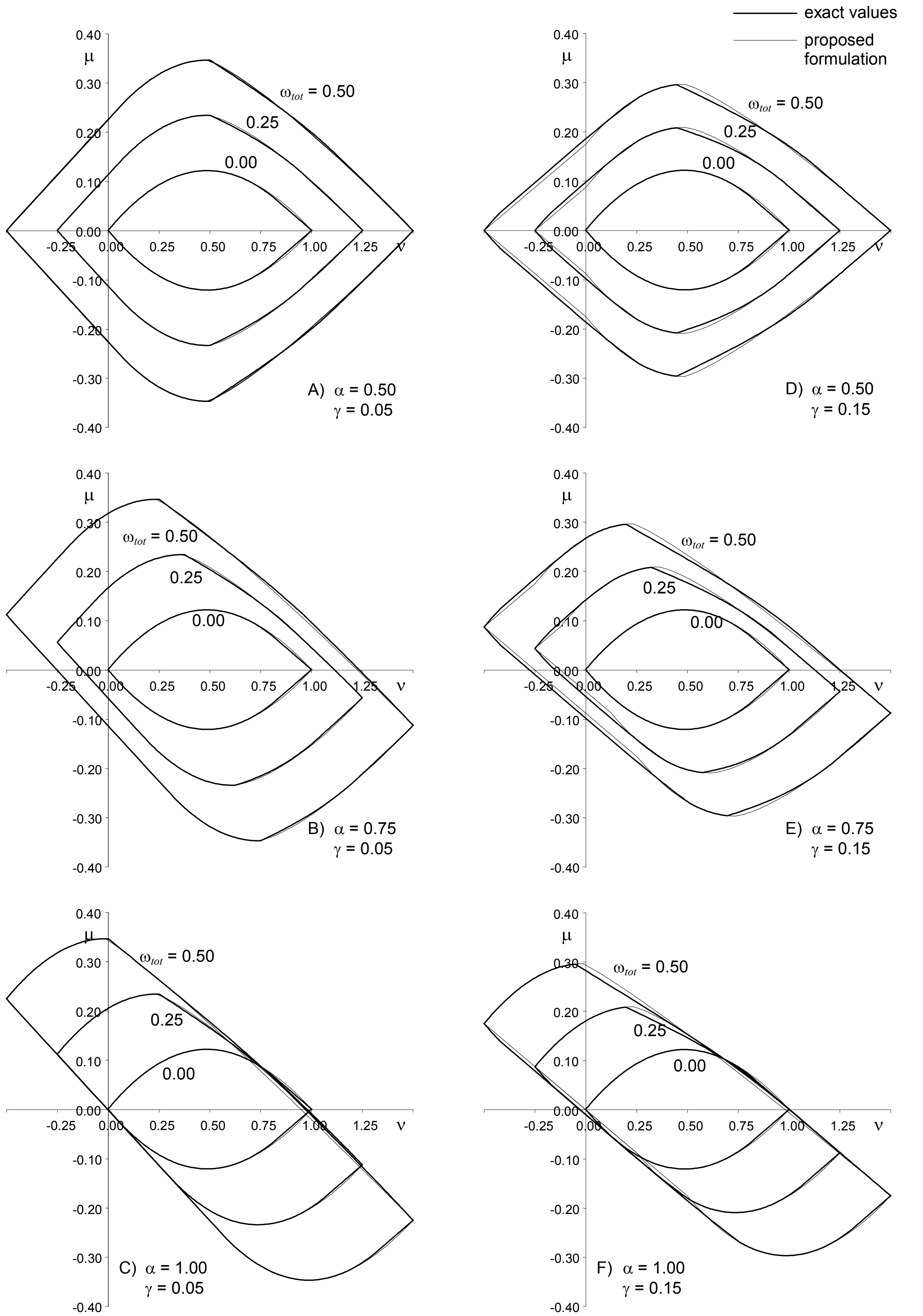

Fig. 6. Interaction curve for asymmetrically reinforced cross section: exact values and proposed formulation (f_{yk} = 430 MPa).

DESIGN FORMULATIONS

While in the case of symmetrical reinforcement simple equations to design cross section dimensions and reinforcement are available [5, 6], if an asymmetrical reinforcement is disposed the number of possible design alternatives increases, making not easy to suggest design procedures and related analytical expressions. When bending moment is the more relevant internal action, an approach similar to the one commonly adopted for section in pure bending may be suggested. Inverting the definition of normalised bending moment, it is possible to obtain the expression

$$h = r'' \sqrt{\frac{M}{b}} \qquad \text{with} \qquad r'' = \frac{1}{\sqrt{\mu\, f_{cd}}} \tag{20}$$

The coefficient r'' has to be evaluated using equations (5) and depends on concrete strength f_{cd}, normalized axial force ν, mechanical reinforcement ratio ω_{tot}, reinforcement distribution (value of α) and, in a minor way, also on normalized cover γ and steel strength f_{yd}. A possible design approach is to define firstly the distribution of reinforcement (coefficient α) and to chose ν and ω_{tot} so as to obtain, using Tab. 3, the most appropriate depth h of the cross section. As an alternative, it is possible to define a priori, basing on experience or on code provisions, a target value of ν and to chose successively proper values of ω_{tot} and α. In this case it should be better to use tables with the same values order by ν instead of α.

Tab. 3A. Coefficient r'' for rectangular cross sections: f_{cd} = 11 MPa, f_{yd} = 379 MPa, γ = 0.1, α = 0.00

α	ν	ω_{tot}						
		0.0	0.1	0.2	0.3	0.4	0.5	0.6
0.00	0.0	-	0.1472	0.1430	0.1415	0.1408	0.1403	0.1400
	0.1	0.0450	0.0450	0.0450	0.0450	0.0450	0.0450	0.0450
	0.2	0.0338	0.0328	0.0327	0.0327	0.0327	0.0327	0.0327
	0.3	0.0296	0.0276	0.0271	0.0270	0.0270	0.0270	0.0270
	0.4	0.0278	0.0251	0.0239	0.0235	0.0235	0.0235	0.0235
	0.5	0.0273	0.0240	0.0222	0.0213	0.0211	0.0211	0.0211
	0.6	0.0281	0.0237	0.0214	0.0201	0.0195	0.0193	0.0193
	0.7	0.0304	0.0242	0.0212	0.0195	0.0185	0.0180	0.0179
	0.8	0.0357	0.0256	0.0216	0.0194	0.0181	0.0173	0.0169
	0.9	0.0504	0.0286	0.0226	0.0196	0.0180	0.0169	0.0162
	1.0	-	0.0346	0.0245	0.0204	0.0182	0.0168	0.0159

Tab. 3B. Coefficient r'' for rectangular cross sections: f_{cd} = 11 MPa, f_{yd} = 379 MPa, γ = 0.1, α = 0.25

α	ν	ω_{tot}						
		0.0	0.1	0.2	0.3	0.4	0.5	0.6
0.25	0.0	-	0.0611	0.0451	0.0374	0.0327	0.0294	0.0270
	0.1	0.0450	0.0374	0.0327	0.0294	0.0270	0.0250	0.0235
	0.2	0.0338	0.0297	0.0270	0.0251	0.0235	0.0222	0.0210
	0.3	0.0296	0.0261	0.0239	0.0223	0.0211	0.0201	0.0193
	0.4	0.0278	0.0244	0.0222	0.0206	0.0195	0.0186	0.0179
	0.5	0.0273	0.0237	0.0214	0.0198	0.0185	0.0176	0.0169
	0.6	0.0281	0.0241	0.0213	0.0194	0.0181	0.0171	0.0162
	0.7	0.0304	0.0251	0.0220	0.0198	0.0181	0.0168	0.0159
	0.8	0.0357	0.0272	0.0231	0.0205	0.0186	0.0171	0.0159
	0.9	0.0504	0.0314	0.0250	0.0216	0.0193	0.0177	0.0164
	1.0	-	0.0401	0.0283	0.0232	0.0203	0.0184	0.0169

Tab. 3C. Coefficient r'' for rectangular cross sections: f_{cd} = 11 MPa, f_{yd} = 379 MPa, γ = 0.1, α = 0.50

α	ν	ω_{tot}						
		0.0	0.1	0.2	0.3	0.4	0.5	0.6
0.50	0.0	-	0.0452	0.0327	0.0270	0.0235	0.0211	0.0193
	0.1	0.0450	0.0327	0.0270	0.0235	0.0211	0.0193	0.0179
	0.2	0.0338	0.0276	0.0239	0.0213	0.0195	0.0180	0.0169
	0.3	0.0296	0.0251	0.0222	0.0201	0.0185	0.0173	0.0162
	0.4	0.0278	0.0240	0.0214	0.0195	0.0181	0.0169	0.0159
	0.5	0.0273	0.0239	0.0214	0.0196	0.0181	0.0170	0.0160
	0.6	0.0281	0.0248	0.0223	0.0204	0.0189	0.0176	0.0166
	0.7	0.0304	0.0264	0.0236	0.0215	0.0198	0.0184	0.0173
	0.8	0.0357	0.0293	0.0255	0.0229	0.0209	0.0194	0.0181
	0.9	0.0504	0.0352	0.0287	0.0250	0.0225	0.0206	0.0190
	1.0	-	0.0497	0.0349	0.0284	0.0247	0.0222	0.0203

Tab. 3D. Coefficient r'' for rectangular cross sections: f_{cd} = 11 MPa, f_{yd} = 379 MPa, γ = 0.1, α = 0.75

α	ν	ω_{tot}						
		0.0	0.1	0.2	0.3	0.4	0.5	0.6
0.75	0.0	-	0.0375	0.0270	0.0223	0.0195	0.0176	0.0162
	0.1	0.0450	0.0296	0.0239	0.0206	0.0185	0.0171	0.0159
	0.2	0.0338	0.0261	0.0222	0.0198	0.0181	0.0168	0.0160
	0.3	0.0296	0.0244	0.0214	0.0194	0.0182	0.0174	0.0167
	0.4	0.0278	0.0237	0.0215	0.0201	0.0191	0.0182	0.0175
	0.5	0.0273	0.0244	0.0226	0.0212	0.0201	0.0192	0.0184
	0.6	0.0281	0.0258	0.0240	0.0226	0.0214	0.0204	0.0195
	0.7	0.0304	0.0281	0.0261	0.0245	0.0231	0.0219	0.0209
	0.8	0.0357	0.0321	0.0294	0.0272	0.0254	0.0239	0.0226
	0.9	0.0504	0.0411	0.0354	0.0317	0.0289	0.0268	0.0251
	1.0	-	0.0727	0.0507	0.0410	0.0353	0.0315	0.0287

Tab. 3E. Coefficient r'' for rectangular cross sections: f_{cd} = 11 MPa, f_{yd} = 379 MPa, γ = 0.1, α = 1.00

α	ν	ω_{tot}						
		0.0	0.1	0.2	0.3	0.4	0.5	0.6
1.00	0.0	-	0.0327	0.0239	0.0201	0.0181	0.0170	0.0168
	0.1	0.0450	0.0276	0.0222	0.0195	0.0182	0.0179	0.0176
	0.2	0.0338	0.0251	0.0214	0.0196	0.0192	0.0189	0.0186
	0.3	0.0296	0.0240	0.0215	0.0208	0.0204	0.0201	0.0199
	0.4	0.0278	0.0240	0.0228	0.0222	0.0218	0.0216	0.0214
	0.5	0.0273	0.0253	0.0244	0.0240	0.0237	0.0235	0.0233
	0.6	0.0281	0.0272	0.0267	0.0264	0.0262	0.0261	0.0260
	0.7	0.0304	0.0302	0.0301	0.0301	0.0300	0.0300	0.0299
	0.8	0.0357	0.0360	0.0362	0.0364	0.0366	0.0367	0.0368
	0.9	0.0504	0.0514	0.0521	0.0527	0.0531	0.0535	0.0540
	1.0	-	-	-	-	-	-	-

NOTATION

A_c	=	$b \times h$ = area of concrete cross section
A_s	=	area of steel reinforcement in the lower portion of rectangular cross section
A'_s	=	area of steel reinforcement in the upper portion of rectangular cross section
$A_{s,tot}$	=	$A_s + A'_s$ = area of total steel reinforcement
b	=	width of rectangular cross section
c	=	concrete cover used in the analysis (more precisely, distance from extreme fibre to centroid of steel bars)
d	=	$h - c$ = effective depth of the cross section
E_s	=	elastic modulus for steel
f_{ck}	=	characteristic cylinder compressive strength of concrete
f_{cd}	=	design value of concrete cylinder compressive strength, for long period uniaxial load
f_{yd}	=	design yield strength of reinforcement
h	=	depth of the rectangular cross section
M	=	bending moment
M_{Rd}	=	design value of resisting bending moment
M_{Ed}	=	design value of acting bending moment
n	=	exponent in the stress-strain relationship for concrete
N	=	axial force (positive if tension)
N_{Rd}	=	design value of resisting axial force (positive if tension)
N_{Ed}	=	design value of acting axial force (positive if tension)
s	=	σ_s / f_{yd} = normalised axial stress of lower steel reinforcement (positive if tension)
s'	=	$-\sigma'_s / f_{yd}$ = normalised axial stress of upper steel reinforcement (positive if compression)
x	=	distance of the neutral axis from the compressed edge
α	=	$A_s / A_{s,tot}$ = normalised area of lower steel reinforcement
α'	=	$A'_s / A_{s,tot}$ = normalised area of upper steel reinforcement ($\alpha' = 1 - \alpha$)
γ	=	c / h = normalised concrete cover
$\Delta\nu_s$	=	variation of normalised axial force corresponding to maximum bending moment, due to the asymmetry of reinforcement
$\Delta\mu_s$	=	variation of bending moment, due to the asymmetry of reinforcement
ε	=	axial strain (positive if elongation)
ε_{c2}	=	limit strain for concrete, for uniformly compressed cross sections ($\varepsilon_{c2} = -2.0 \times 10^{-3}$ if $f_{ck} \leq$ 50 MPa)
$\varepsilon_{c,\min}$	=	axial strain at the less compressed edge (for fully compressed cross sections)
ε_{cu2}	=	ultimate strain for concrete ($\varepsilon_{cu2} = -3.5 \times 10^{-3}$ if $f_{ck} \leq 50$ MPa)
ε_O	=	axial strain at the origin O
ε_{yd}	=	f_{yd} / E_s = yield strain for steel
$\eta_{\min}$	=	$\varepsilon_{c,\min} / \varepsilon_{c2}$ = normalised axial strain at the less compressed edge
$\eta_{\min\,(D')}$	=	value of $\eta_{\min}$ for strain diagram D'
μ	=	$\dfrac{M}{f_{cd}\ A_c\ h}$ = normalised bending moment

$\mu_{c,\max}$ = maximum contribution of concrete to normalised bending moment

$\mu_{s,\max}$ = maximum contribution of total reinforcement to normalised bending moment

ν = $\dfrac{N}{-f_{cd}\ A_c}$ = normalised axial force (positive if compression)

ν_M = normalised axial force corresponding to the maximum contribution of concrete to bending moment

$\nu_{s,\max}$ = maximum contribution of total reinforcement to normalised axial force

σ = axial stress (positive if tension)

σ_s = axial stress of lower steel reinforcement (positive if tension)

σ'_s = axial stress of upper steel reinforcement (positive if tension)

ξ = normalised depth of the compressed part of the cross section ($\xi = x / h$ for partially compressed cross sections, $\xi = 1$ for fully compressed cross sections)

$\xi_1\ \xi_2\ \xi_3$ = normalised distance of neutral axis from the compressed edge of the cross section, for strain diagrams C'_{s+} C'_{s-} and C'

ω_{tot} = $\dfrac{A_{s,tot}}{A_c}\dfrac{f_{yd}}{f_{cd}}$ = mechanical reinforcement ratio

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